Multipole Mixing Ratios of Gamma Rays from the ${}^{58}_{28}\mathrm{Ni}({}^{24}_{12}\mathrm{Mg},\mathrm{pn}\gamma){}^{80}_{39}\mathrm{Y}$ Reaction Using Constant Statistical Tensor Method and Other Related Methods

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ABSTRACT

The multipole mixing ratios (δ-values) of gamma (γ)-transitions from low and high-spin states excited in the ${}^{58}_{28}\mathrm{Ni}({}^{24}_{12}\mathrm{Mg},\mathrm{pn}\gamma){}^{80}_{39}\mathrm{Y}$ reaction are calculated in the present work using the a_2-ratio, constant statistical tensor (CST) and least squares fitting (LSF) methods together with the experimental angular distribution a_2-coefficients reported for such γ-transitions. The good agreement between the δ-values calculated by these methods for most γ-transitions confirm the validity of the three methods in calculating the δ-values of such γ-transitions. The weighted averages of δ-values calculated for mixed γ-transitions are presented as adopted values.

1- INTRODUCTION

Bucureseu et. al. [1] have recently studied the high spin states of the odd-odd nucleus ^{80}Y using the ^{24}Mg(^{58}Ni,pnγ)^{80}Y and ^{58}Ni(^{24}Mg, pnγ)^{80}Y reactions at 180 and 77 MeV beam energies respectively. Gamma-ray transitions in this nucleus have been unambiguously assigned and arranged into several rotational bands extending up to an excitation energy of about 12 MeV and spin J=24. The angular distributions of several γ-rays from the ^{58}Ni(^{24}Mg, pnγ)^{80}Y reaction have been measured but the corresponding δ-mixing ratios were not reported.

The δ-values of γ-transitions from levels of ^{80}Y have not been reported in the Table of Isotopes, 8th edition [2]. This indicates that the δ-values of such transitions have not been determined previously.

In the present work, the angular distribution a_2-coefficients reported in ref. [1] for several γ-transitions have been used to calculate δ-values of the γ-transitions using a_2-ratio, constant statistical tensor (CST) and least squares fitting (LSF) methods. The a_2-ratio method depends only on the experimental a_2-coefficients measured for at least two γ-transitions from the same initial state, one of which is a pure transition or might be considered as a pure E1 or E2 transition.

The CST-method was suggested by Youhana [3,4] who noticed that the magnetic substate population parameters calculated by Ameen [5] as a partial fulfillment of the requirement for the Ph.D. degree under his supervision, were almost constant for levels exited in ^{92,94}Zr(n,n`γ) with the same spin value for both parities.

The program POP used in ref. [5] was a reduced and modified version of the computer code CINDY [6]. It was, therefore, concluded that the population parameters of levels with the same spin value depend neither upon the energy of the level nor upon its parity. Depending upon this fact, Youhana [3,4] has concluded that the statistical tensor, which is related to the population parameters should also be constant for levels

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with the same spin value and as a result, CST-method was suggested as a tool for calculating δ-values of mixed transitions. This method was then applied for the first time by Youhana [3,4] to calculate the δ-values of γ-transitions from levels of ^{90,92,94}Zr and ^{150}Nd excited in the (n,n`γ) reactions. Mohammed-Said [7], in her Ph.D. work under his supervision, has applied this method to calculate the δ-values of γ-transitions from levels excited in $^{23}$Na(α,pγ)$^{26}$Mg, $^{162,164}$Dy(n,n`γ) and ^{168}Er(n,n`γ) reactions. She has also applied it to calculate δ-values of γ-rays in coincidence with proton group leading to an excited state populated in ^{25}Mg(d,pγ)^{26}Mg reaction. In these studies, the validity of this method as a tool as good as the computer code CINDY for calculating δ-values of γ-transitions was not confirmed only but also its capability of predicting any inaccuracy existing in the experimental data.

The LSF-method was used in the present work for the first time to estimate the statistical tensors of all levels particularly those with certain spin values which have no pure transitions. More details are given in the next section.

The main aim of the present work was to confirm the validity of these methods as tools for calculating δ-values of γ-transitions from high-spin states excited in heavy-ion reactions and their capability of predicting any inaccuracy in the experimental data.

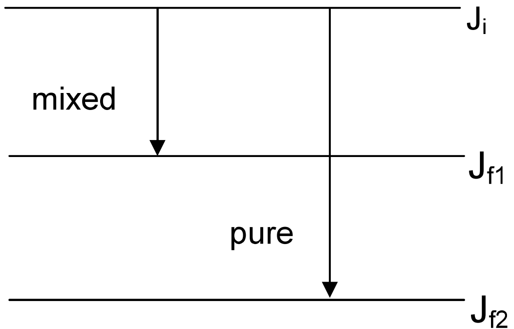

Fig.(1)

2- DATA REDUCTION AND ANALYSIS

A) a_2-Ratio Method

For levels that have at least two γ-transitions, one of which is pure transition, such as that shown in fig. (1), the a_2-coefficient of mixed transitions is related to the statistical tensor $\rho_2(J_i)$ by the following relationship [8].

$$a_2(J_i - J_{f_1}) = \rho_2(J_i)\frac{F_2(J_{f_1}L_1L_1J_i) + 2\delta F_2(J_{f_1}L_1L_2J_i) + \delta^2 F_2(J_{f_1}L_2L_2J_i)}{1+\delta^2} \quad \text{......(1)}$$

For the pure $(J_i - J_{f_2})$ transition, δ is zero and hence:

$$a_2(J_i - J_{f_2}) = \rho_2(J_i)F_2(J_{f_2}L_1L_1J_i) \quad \text{....................(2)}$$

where L_1 is the angular momentum of the γ-transitions given by:

$|J_i - J_f| \le L_1 \le J_i + J_f$ ....(3)

$L_2 = L_1+1$ and $L_1 \neq 0$

Since $\rho_2(J_i)$ is the same for both transitions, then

$$\frac{a_2(J_i - J_{f_1})}{a_2(J_i - J_{f_2})} = \frac{F_2(J_{f_1}L_1L_1J_i) + 2\delta F_2(J_{f_1}L_1L_2J_i) + \delta^2 F_2(J_{f_1}L_2L_2J_i)}{F_2(J_{f_2}L_1L_1J_i)(1+\delta^2)} \quad \text{...(4)}$$

The F_2-coefficients have been calculated by Al-Zuhairy [9] for integral J-values up to J=40 and half-integral J-values up to $J = \frac{61}{2}$. Using the related experimental a_2-coefficients and the related F_2-coefficients, the δ-values of the mixed transitions can be calculated from eq. (4).

B) CST-Method

The a_2-coefficient is, in general, related to the statistical tensor, $\rho_2(J_i)$ by the relationship:

$$a_2(J_i - J_{f_1}) = \rho_2(J_i)\frac{F_2(J_fL_1L_1J_i) + 2\delta F_2(J_fL_1L_2J_i) + \delta^2 F_2(J_fL_2L_2J_i)}{1+\delta^2} \quad \text{...(5)}$$

For pure transitions or transitions considered to be pure, δ=0 and hence:

$$\rho_2(J_i) = \frac{a_2(J_i - J_{f_2})}{F_2(J_fL_1L_1J_i)} \quad \text{............................. (6)}$$

Using the a_2-coefficient reported for such transition, $\rho_2(J_i)$ values can be calculated for all initial levels that have at least one pure or considered to be pure transition. The $\rho_2(J_i)$ values thus calculated are considered to be constant for all levels with the same J_i-values and can then be used in eq.(5) to calculate the δ-values for the γ-transitions considered to be pure and for other mixed transitions.

C) LSF-Method

In this method, the $\rho_2(J_i)$ values calculated for levels with different J_i-values are computer fitted to a polynomial series of the form:

$$\rho_2(J_i) = \sum_{x=0}^{x=n} B_x J_i^x \quad \text{.................................. (7)}$$

using the least squares fitting program that was written in the present work to determine the B_x parameters for n=1,2,3 and 4 and the χ^2-value for each n. The set with minimum χ^2 was then used to calculate the $\rho_2(J_i)$ values for all J_i-values. The $\rho_2(J_i)$ values thus obtained are then used to calculate the δ-values for all γ-transitions whose angular distribution have been measured.

3- RESULTS AND DISCUSSION

Note: In all tables presented in this chapter, the errors in the last figure or figures are given in parentheses unless specified. If the difference between (+ error) and (-

error) is 0.01 for small |δ| values and is 0.1 for large |δ| values, the large error is taken into consideration.

In the present work, the a_2-coefficients reported in ref. [1] for 67 γ-rays assigned to ^{80}Y in the $^{58}Ni(^{24}Mg,pn\gamma)$ reaction are used to calculate the corresponding δ-values by a_2-ratio, CST and LSF-methods as follows:

A- Results calculated by a_2-ratio:

The energy levels of ^{80}Y and the related γ-transitions whose a_2-coefficients have been used to calculate the corresponding δ-values by this method are presented in Table (1). It can be seen in this table that the δ-value calculated for the 802.1 KeV (6^+-5^-) transition from the 1059.5 KeV level assuming pure E1 transition for the 489.9 KeV (6^+-6^-) transition from the same level is more likely than that calculated for the 489.9 KeV transition assuming pure E1 transition for the 802.1 KeV. This indicates that $a_2(489.9)$ is better than $a_2(802.1)$ for calculating $\rho_2(6)$.

In the case of the 1490.0 KeV level, both $a_2(553.0)$ and $a_2(315.0)$ are good enough to be used to calculate $\rho_2(8)$. The 304.7 KeV (8^+-6^+) transition was not taken into consideration due to large error associated with the corresponding a_2-coefficient.

The imaginary roots obtained in calculating the δ-values of the 474.2 KeV (8^+-7^+) transition from the 1764.1 KeV level indicate that the a_2-coefficient reported in ref. [1] for this transition is not accurately measured. The other γ-transitions can not discussed at this stage since no δ-values are reported in ref. [1].

Table (1)

Multipole mixing ratios of γ-transition from energy levels of ^{80}Y calculated by the a_2-ratio method.

E_{level} KeV	E_γ KeV	$J_i^\pi - J_f^\pi$	a_2[1]	a_4[1]	δ	
					a_2-Ratio	
570.0	570.1	6^--4^-	0.12(5)	0.06(8)	E2	
	312.9	6^--5^-	-0.17(5)	0.13(10)	$-(0.16^{+0.22}_{-0.16})$ $-(3.2^{+4.2}_{-1.4})$	
648.1	336.1	4^+-2^+	0.18(12)	0.10(16)	E2	
	192.9	4^+-3^+	-0.09(3)	-0.02(3)	0.05(9) $-(7^{+12}_{-3})$	
663.2	339.7	4^--2^-	0.19(9)	-0.04(13)	E2	
	203.0	4^--3^-	-0.17(5)	-0.05(8)	-0.05(12) $-(4.2^{+4.3}_{-1.6})$	
878.2	418.0	5^--3^-	0.28(14)	0.07(21)	E2	
	215.1	5^--4^-	-0.18(5)	0.02(9)	0.01(9) $-(6.9^{+9.3}_{-2.6})$	
1059.5	802.1	6^+-5^-	-0.28(8)	0.16(13)	E1	-0.13(10)
	489.9	6^+-6^-	0.24(3)	-0.03(7)	-0.32(11)	E1
1175.2	527.1	6^+-4^+	0.25(10)	0.05(16)	E2	
	289.0	6^+-5^+	-0.04(4)	0.01(7)	0.12(4) Only	
1185.2	537.0	6^+-4^+	0.20(10)	-0.02(17)	E2	
	299.2	6^+-5^+	-0.05(4)	0.02(8)	0.10(5) Only	

E_{level} KeV	E_γ KeV	$J_i^\pi - J_f^\pi$	$a_2[1]$	$a_4[1]$	δ	
					a_2-Ratio	
1490.2	553.0	8^+-7^-	-0.15(4)	0.10(12)	0.00(1)	E1
	315.0	8^+-6^+	0.22(9)	0.23(15)	E2	0.01(17)
	304.7	8^+-6^+	0.17(22)	-0.01(36)	$-(0.07^{+0.41}_{-0.38})$	-0.07(40)
	200	8^+-7^+	-0.59(4)	0.06(8)	$-(0.61^{+?}_{-0.42})$ $-(1.3^{+1.9}_{-?})$	$-(0.68^{+?}_{-0.37})$ $-(1.2^{+1.1}_{-?})$
1509.0	623.0	7^+-5^+	0.23(13)	-0.01(16)	E2	
	333.7	7^+-6^+	-0.38(4)	0.14(7)	$-(0.22^{+0.37}_{-0.22})$ $-(2.9^{+5.2}_{-1.6})$	
	323.9	7^+-6^+	-0.26(11)	0.10(21)	$-(0.09^{+0.21}_{-0.17})$ $-(4.6^{+18.1}_{-2.3})$	
1764.1	588.9	8^+-6^+	0.26(6)	-0.09(9)	E2	
	474.2	8^+-7^+	-0.93(6)	0.21(9)	Imaginary Roots	
1823.7	533.7	9^+-7^+	0.34(11)	-0.14(11)	E2	
	335.5	9^+-8^+	-0.51(5)	0.07(10)	$-(0.17^{+0.13}_{-0.11})$ $-(3.6^{+2.5}_{-1.2})$	
1956.7	750.2	8^--6^-	0.28(5)	-0.08(9)	E2	
	468.6	8^--7^-	-0.11(6)	0.23(10)	0.06(5) $-(21^{+\infty}_{-10})$	
2615.2	791.5	11^+-9^+	-0.25(6)	0.01(10)	E2	
	347.0	11^+-10^+	-0.46(6)	0.15(10)	$-(0.25^{+0.15}_{-0.12})$ $-(2.9^{+1.5}_{-0.9})$	

B- Results calculated by the CST-method:

The energy levels of ^{80}Y and the related γ-transitions whose a_2-coefficients have been used to calculate the $\rho_2(J_i)$ values are presented in Table (2) together with the $\rho_2(J_i)$ values and the weighted averages. The $\rho_2(J_i)$ values marked with (*) are not used in the calculation of weighted averages for several reasons such as, the errors associated with the a_2-coefficients are relatively large, the γ-transition has two placements and the related a_2-coefficients are, clearly, under -or over- estimated.

Table (2)

Levels and γ-transitions of ^{80}Y used to calculate $\rho_2(J_i)$

E_{level} KeV	E_γ KeV	$J_i^\pi - J_f^\pi$	$a_2[1]$	$a_4[1]$	$\rho_2(J_i)$	Weighted Average
648.1	336.1	4^+-2^+	0.18(12)	0.10(16)	-0.40205(26804)	-0.41635(16082)
663.2	339.7	4^--2^-	0.19(9)	-0.04(13)	-0.42439(20103)	
878.2	418.0	5^--3^-	0.28(14)	0.07(21)	-0.66578(33289)	-0.66578(33289)
570.0	570.1	6^--4^-	0.12(5)	0.06(8)	-0.29783(12410)*	-0.54371(6316)
1059.0	802.1	6^+-5^-	-0.28(8)	0.16(13)	-0.99277(28365)*	
	489.9	6^+-6^-	0.24(3)	-0.03(7)	-0.54152(6769)	
1175.2	527.1	6^+-4^+	0.25(10)	0.05(16)	-0.62049(24819)	
1185.2	537.0	6^+-4^+	0.20(10)	-0.02(17)	-0.49639(24819)	
937.6	680.6	7^--5^-	0.19(3)	-0.05(5)	-0.48654(7682)	-0.49172(7485)

E_{level} KeV	E_γ KeV	$J_i^\pi - J_f^\pi$	a_2[1]	a_4[1]	$\rho_2(J_i)$	Weighted Average
1509.0	623.0	7^+-5^+	0.23(13)	-0.01(16)	-0.58897(33290)	
1358.6	788.4	8^--6^-	0.24(5)	-0.03(7)	-0.62939(13112)	-0.64889(6754)
1490.2	553.0	8^+-7^-	-0.15(4)	0.10(12)	-0.56195(14985)	
	315.0	8^+-6^+	0.22(9)	0.23(15)	-0.57694(23602)	
	304.7	8^+-6^+	0.17(22)	-0.01(36)	-0.44582(57694)*	
1764.1	588.9	8^+-6^+	0.26(6)	-0.09(9)	-0.68184(15735)	
1915.7	730.5	8^+-6^+	0.14(19)	-0.24(30)	-0.36715(49827)*	
1956.7	750.2	8^--6^-	0.28(5)	-0.08(9)	-0.73429(13112)	
1823.7	533.7	9^+-7^+	0.34(11)	-0.14(11)	-0.90851(29393)	-0.65980(7734)
1825.2	887.6	9^--7^-	0.24(3)	-0.17(5)	-0.64130(8016)	
2322.7	813.8	9^+-7^+	0.21(32)	-0.31(51)	-0.56114(85507)*	
2267.5	777.3	10^+-8^+	0.29(3)	-0.10(5)	-0.78674(8139)	-0.79241(6272)
2351.0	992.3	10^--8^-	0.26(6)	0.24(9)	-0.70535(16277)	
2618.0	853.9	10^+-8^+	0.29(6)	-0.24(9)	-0.78674(16277)	
2915.7	959.0	10^--8^-	0.35(7)	-0.01(8)	-0.94951(18990)	
2615.2	791.5	11^+-9^+	0.25(6)	0.01(10)	-0.68676(16482)*	-0.88919(15753)
2901.2	1076.0	11^--9^-	0.49(4)	0.20(7)	-1.34604(10988)*	
3327.7	1005.0	11^+-9^+	0.27(10)	0.01(15)	-0.74170(27470)	
3332.4	1027.4	11^--9^-	0.35(7)		-0.96146(19229)	
3298.3	1030.6	12^+-10^+	0.39(7)	0.04(9)	-1.08264(19432)	-1.02430(17968)
3531.1	1180.1	12^--10^-	0.35(8)	-0.14(12)	-0.97160(22203)*	
3689.2	1071.2	12^+-10^+	0.28(7)	-0.26(26)	-0.77728(47192)	
3628.0	1012.8	13^+-11^+	0.46(6)	0.11(8)	-1.28894(16805)	-1.04067(10758)
4146.1	1244.9	13^--11^-	0.31(5)	0.08(8)	-0.86827(14004)	
4442.1	1109.7	13^--11^-	0.24(6)	-0.30(9)	-0.67221(16805)*	
5193.4	1180.1	13^+-11^+	0.35(8)	-0.14(12)	-0.98031(22407)*	
4558.6	1260.3	14^+-12^+	0.36(13)	-0.06(19)	-1.01609(36692)	-1.16949(31589)
4842.6	1311.5	14^--12^-	0.20(8)	0.06(13)	-0.56449(22580)*	
4973.2	1284.0	14^+-12^+	0.57(22)	0.09(34)	-1.60881(62094)	
4848.3	1220.3	15^+-13^+	0.30(6)	0.10(9)	-0.85244(17049)	-0.99035(13460)
5446.7	1300.6	15^--13^-	0.45(9)	0.09(12)	-1.27866(25573)	
?	1457.5	15-13	0.37(15)	0.19(23)	-1.05135(42622)	
5998.7	1440.1	16^+-14^+	0.45(8)	0.18(11)	-1.28619(22866)	-1.29228(22043)
2097.6	1255.0	16^--14^-	0.48(29)	-0.17(36)	-1.37194(82888)	
7450.2	1352.6	18^--16^-	0.27(14)	-0.20(22)	-0.77936(40411)	-0.77936(40411)

The $\rho_2(J_i)$ values thus obtained were then used to calculate the δ-values of all γ-transitions whose angular distributions have been measured. The results are presented in Table (3). As it can be seen, the δ-values of γ-transitions from levels with J_i=2 and J_i=3 could not be calculated by this method since no pure transitions from such levels were reported in ref. [1]. The results of γ-transitions from other levels are discussed as follows:

(1) The levels with J_i=4

The δ-values calculated for (4-2) transitions are consistent with the pure E2 transition expected for such γ-transitions. The δ-values calculated for the 192.9 and 203.0 (4^--3^-) KeV (4^+-3^+) transition from the 648.1 and 663.2 KeV levels respectively are in rather good agreement with those calculated by the a_2-ratio method.

(2) The levels with J_i=5

One pure transition, namely, the 418.0 KeV (5^--3^-) transition from the 878.2 KeV level, was reported in ref. [1] only. The a_2-coefficient reported for this transition was used to calculate $\rho_2(5)$ which was then used to calculate the δ-values of other γ-transitions from levels with J_i=5. These values can not be further discussed at this stage since no δ-values to be compared with the values of the present work were reported in ref. [1].

(3) The levels with J_i=6

The δ-values calculated for the 527.1 (6^+-4^+) and 537.0 KeV(6^+-4^+) transitions from the 1175.2 and 1185.2 KeV levels respectively are consistent with the pure E2 transitions expected for such γ-transitions. The δ-value calculated for the 489.9 KeV(6^+-6^-) transition from the 1059.5 KeV level is also consistent with pure E1 transition, whereas the |δ|-value calculated for the 570.1KeV(6^--4^-) transition to the ground state is slightly larger than that expected for pure E2 transitions. This indicates that the a_2-coefficient reported in ref. [1] for this transition is not accurate. The δ-values calculated for other γ-transitions from levels with J_i=6 can not be discussed for the same reason mentioned in (2).

(4) The levels with J_i=7

The δ-values calculated for the 680.6 (7^--5^-) and 623.0KeV(7^+-5^+) transitions from the 937.0 and 1509.0 KeV levels respectively but the first one seems to be better than the second one due to the relatively large error associated with the a_2-coefficient of the second transition. No further discussion can be made for the δ-values calculated for the γ-transitions from levels with J_i=7 at this stage.

(5) The levels with J_i=8

The δ-values calculated for the 788.4 (8^--6^-) and 315.0 (8^+-6^+), 588.9 (8^+-6^+) and 750.2 KeV(8^--6^-) transitions from the 1358.6, 1490.2, 1764.1 and 1956.7 KeV levels respectively are consistent with the pure E2 transitions expected for such γ-transitions. In addition, the δ-value calculated for the 553.0 KeV(8^+-7^-) transition from the 1490.2 KeV level is consistent with the pure E1 transition, whereas the δ-values calculated for the 304.7 and 730.5 KeV(8^+-6^+) transitions from the 1490.2 and 1915.7 KeV levels indicate that the a_2-coefficients reported in ref. [1] for both transitions are inaccurate. This is obvious from the relatively large errors associated with both a_2-coefficients. The imaginary roots obtained in calculating the δ-values of the 474.2 KeV transition from the 1764.1 KeV level confirm the inaccuracy in the a_2-coefficient reported in ref. [10] for this transition.

(6) The levels with J_i=9

The δ-value calculated for the 887.6 KeV (9^--7^-) transition from the 1825.3 KeV level confirms the pure E2 transition expected for such γ-transition. The δ-value of the 813.8 KeV(9^+-7^+) transition from the 2322.7 KeV level is also consistent with the pure E2 transition but the associated errors are rather large, whereas that of the 533.7 KeV(9^+-7^+) transition from the 1823.7 KeV level is slightly larger than that expected for a pure E2 transition. The δ-values calculated for other γ-transitions from levels with J_i=9 can not be discussed at this stage.

(7) The levels with J_i=10

The δ-values calculated for all (10-8) transitions are in a good agreement with that expected for a pure E2 transition but the a_2-coefficient reported in ref. [1] for the 959.0 KeV(10^--8^-) transition from the 2915.7 KeV level seems to be slightly over-estimated.

(8) The levels with J_i=11

The δ-value calculated for the 791.5 KeV(11^+-9^+) transition from the 2615.2 KeV level is in a fair agreement with that expected for a pure E2 transition. The δ-values calculated for the 347.0 KeV(11^+-10^+) transition from the same level are, within the errors, in a fair agreement with those calculated by the a_2-ratio method but the values themselves are different. This indicates that the a_2-coefficient reported in ref. [1] for the 791.5 KeV(11^+-9^+) transition is under-estimated. The δ-value calculated for the 1076.0 KeV(11^--9^-) transition from the 2901.2 KeV level indicates that the corresponding a_2-coefficient is over-estimated, whereas that calculated for the 1005.0 KeV(11^+-9^+) transition from the 3327.7 KeV level is in agreement with that expected for a pure E2 transition.

(9) The levels with J_i=12

The δ-values calculated for the 1030.6 (12^+-10^+) and 1180.1KeV (12^--10^-) from the 3298.3 and 3531.1 KeV levels respectively are in a good agreement with that expected for a pure E2 transition, whereas that of the 1071.2 KeV(12^+-10^+) transition from the 3689.2 KeV level indicates that the a_2-coefficient reported in ref. [1] for this transition is under-estimated. The inaccuracy of this coefficient is obvious from the relatively large error associated with it. It should be noted that the 1180.1 KeV transition has also been assigned in ref. [1] as a (13^+-11^+) transition from the 5193.4 KeV level.

(10) The levels with J_i=13

The δ-value calculated for the 1180.1 KeV(13^+-11^+) transition from the 5193.4 KeV level is also in good agreement with that expected for a pure E2 transition and hence no distinction can be made between the two placements. The δ-value calculated for the 1244.9 KeV(13^--11^-) transition from the 4146.1 KeV level is also consistent with that of a pure E2 transition, whereas those calculated for the 1012.8 (13^+-11^+) and 1109.7 KeV(13^--11^-)transitions from the 3628.0 and 4442.1 KeV levels respectively indicate that the a_2-coefficient reported in ref. [1] for the 1012.8 KeV transition is over-estimated and that reported for the 1109.7 KeV transition is under-estimated.

(11) The levels with J_i=14

The δ-value calculated for the 1260.3 KeV (14^+-12^+) transition from the 4558.0 KeV level is consistent with that expected for a pure E2 transition, whereas those calculated for the 1311.5 (14^--12^-) and 1284.0 KeV(14^+-12^+)transitions from the 4842.6 and 4973.2 KeV levels respectively, indicate that the a_2-coefficient of the first transition is under-estimated and that of the second transition is over-estimated.

(12) The levels with J_i=15

The δ-value calculated for the 1220.3 KeV(15^+-13^+) transition from the 4848.3 KeV level is consistent with that expected for a pure E2 transition, whereas that of the 1300.6 KeV(15^--13^-)transition from the 5446.7 KeV level indicates that the a_2-coefficients reported for this transition is over-estimated.

A γ-transition of energy 1457.5 KeV has been reported in ref. [1] but it has not been assigned to the decay of any level in ^{80}Y. Its angular distribution has been measured and it has been assigned as a (15-13) transition. The δ-value calculated for this transition has relatively large error but the value itself is in a good agreement with that expected for a pure E2 transition.

(13) The levels with J_i=16

The δ-values calculated for the 1440.1 (16^+-14^+) and 1255.0KeV(16^--14^-) transitions from the 5998.7 and 6097.6 KeV levels respectively are in a good agreement with that expected for a pure E2 transition.

(14) The levels with J_i=17

Only one (18-16) transition has been reported in ref. [1] and hence the δ-value calculated for this transition, namely, 1352.6 KeV(18^--16^-) transition from the 7450.2

KeV level, should be zero since its a_2-coefficient was used to calculate $\rho_2(18)$. Therefore, no further discussion can be made at this stage.

Table (3)

Multipole mixing ratios of γ-transitions from levels of ^{80}Y calculated by CST and LSF methods

E_{level} KeV	E_γ KeV	$J_i^\pi - J_f^\pi$	a_2[1]	a_4[1]	δ	
					CST	LSF
0.0	-	4^-				
257.0	257.0	5^--4^-	-0.12(3)	0.06(5)	0.06(5) $-(10^{+13}_{-4})$	0.02(4) $-(7.2^{+2.5}_{-1.4})$
323.6	95.5	2^--1^-	-0.12(3)	-0.07(6)	-	0.04(5) -3.0(5)
455.2	143.0	3^+-2^+	-0.14(2)	-0.07(3)	-	-0.01(3) -3.8(5)
459.8	132.6	3^--2^-	-0.16(4)	-0.06(6)	-	-0.04(6) $-(3.4^{+0.9}_{-0.6})$
570.0	570.1	6^--4^-	0.12(5)	0.06(8)	-0.16(9)	-0.15(8)
	312.9	6^--5^-	-0.17(5)	0.13(10)	-0.02(6) $-(6.5^{+3.5}_{-1.8})$	-0.02(6) $-(6.2^{+3.0}_{-1.6})$
648.1	336.1	4^+-2^+	0.18(12)	0.10(16)	$-(0.01^{+0.36}_{-0.33})$	$-(0.01^{+0.31}_{-0.29})$
	192.9	4^+-3^+	-0.09(3)	-0.02(3)	0.05(6) $-(7.3^{+\ .3}_{-2.2})$	0.05(4) $-(7.3^{+2.7}_{-1.6})$
663.2	339.7	4^--2^-	0.19(9)	-0.04(13)	0.01(29)	0.01(21)
	203.0	4^--3^-	-0.17(5)	-0.05(8)	-0.05(11) $-(4.1^{+3.3}_{-1.4})$	-0.05(7) $-(4.1^{+1.}_{-\ .})$
878.2	418.0	5^--3^-	0.28(14)	0.07(21)	0.00(29)	$0.17^{+\ .45}_{-\ .8}$
	215.1	5^--4^-	-0.18(5)	0.02(9)	0.01(8) $-(6.9^{+\ .3}_{-\ .})$	-0.05(6) $-(4.8^{+1.}_{-1.1})$
886.0	237.9	5^+-4^+	-0.14(3)	0.00(5)	0.04(7) $-(8.9^{+10.6}_{-3.2})$	0.00(4) $-(6.2^{+1.7}_{-1.2})$
937.6	680.6	7^--5^-	0.19(3)	-0.05(5)	0.00(8)	-0.06(4)
1059.5	802.1	6^+-5^-	-0.28(8)	0.16(13)	-0.13(10)	-0.14(10)
	489.9	6^+-6^-	0.24(3)	-0.03(7)	$0.00^{+?}_{-0.1}$	$0.04^{+?}_{-0.12}$
1175.2	527.1	6^+-4^+	0.25(10)	0.05(16)	0.05(18)	$0.07^{+0.25}_{-0.17}$
	289.0	6^+-5^+	-0.04(4)	0.01(7)	0.11(4) only	0.11(4) only
1185.2	537.0	6^+-4^+	0.20(10)	-0.02(17)	-0.03(17)	-0.02(17)
	299.2	6^+-5^+	-0.05(4)	0.02(8)	0.10(4) only	0.10(4) only
1206.5	328.2	6^--5^-	-0.22(3)	-0.0(5)	-0.07(4) $-(4.8^{+1.}_{-0.})$	-0.08(3) $-(4.6^{+0.}_{-0.})$

E_{level} KeV	E_γ KeV	$J_i^\pi - J_f^\pi$	$a_2[1]$	$a_4[1]$	δ	
					CST	LSF
1290.0	231.0	7^+-6^+	-0.36(5)	0.04(9)	$-(0.28^{+0.13}_{-0.10})$ $-(2.4^{+0.}_{-0.})$	-0.20(6) -3.1(6)
1358.6	788.4	8^--6^-	0.24(5)	-0.03(7)	-0.01(7)	-0.02(7)
1488.1	281.4	7^--6^-	-0.23(3)	0.07(5)	-0.11(5) $-(4.3^{+1.3}_{-0.9})$	-0.06(3) $-(5.3^{+1.0}_{-0.7})$
1490.2	553.0	8^+-7^-	-0.15(4)	0.10(12)	0.02(4)	0.02(4)
	315.0	8^+-6^+	0.22(9)	0.23(15)	-0.04(12)	-0.04(12)
	304.7	8^+-6^+	0.17(22)	-0.01(36)	$-(0.10^{+0.32}_{-0.28})$	$-(0.11^{+0.33}_{-0.30})$
	200.0	8^+-7^+	-0.59(4)	0.06(8)	$-(0.44^{+0.17}_{-0.10})$ -1.7(4)	-0.42(7) -1.7(2)
1509.0	623.0	7^+-5^+	0.23(13)	-0.01(16)	$0.07^{+0.29}_{-0.2}$	0.00(20)
	333.7	7^+-6^+	-0.38(4)	0.14(7)	$-(0.31^{+0.13}_{-0.10})$ $-(2.2^{+0.8}_{-0.})$	-0.22(5) -2.9(4)
	323.9	7^+-6^+	-0.26(11)	0.10(21)	-0.14(16) $-(3.7^{+3.9}_{-1.})$	-0.09(12) $-(4.6^{+4.3}_{-1.6})$
1764.1	588.9	8^+-6^+	0.26(6)	-0.09(9)	0.02(8)	0.01(8)
	474.2	8^+-7^+	-0.93(6)	0.21(9)	Imaginary Roots	Imaginary Roots
1823.7	533.7	9^+-7^+	0.34(11)	-0.14(11)	$0.12^{+0.19}_{-0.1}$	0.07(14)
	335.5	9^+-8^+	-0.51(5)	0.07(10)	-0.32(10) -2.3(6)	-0.26(5) -2.7(4)
1825.3	887.6	9^--7^-	0.24(3)	-0.17(5)	-0.01(5)	-0.04(4)
1915.7	730.5	8^+-6^+	0.14(19)	-0.24(30)	$-(0.14^{+0.28}_{-0.24})$	$-(0.15^{+0.29}_{-0.2})$
1956.7	750.2	8^--6^-	0.28(5)	-0.08(9)	0.04(7)	$0.06^{+0.04}_{-0.09}$
	468.6	8^--7^-	-0.11(6)	0.23(10)	0.05(6) $-(17^{+\infty}_{-12})$	0.06(5) $-(17.9^{+\infty}_{-8.4})$
2267.5	777.3	10^+-8^+	0.29(3)	-0.10(5)	0.00(4)	-0.01(4)
2305.0	348.4	9^--8^-	-0.24(6)	-0.09(9)	-0.06(6) $-(6.2^{+.4}_{-.})$	-0.03(5) $-(7.3^{+.}_{-.})$
2322.7	813.8	9^+-7^+	0.21(32)	-0.31(51)	$-(0.05^{+.}_{-.4})$	$-(0.08^{+.4}_{-.4})$
2351.0	992.3	10^--8^-	0.26(6)	0.24(9)	-0.03(7)	-0.05(7)
2615.2	791.5	11^+-9^+	0.25(6)	0.01(10)	-0.07(7)	-0.07(6)
	347.0	11^+-10^+	-0.46(6)	0.15(10)	-0.15(7) $-(4.1^{+.}_{-.})$	-0.15(4) $-(4.2^{+.}_{-.})$
2618.0	853.9	10^+-8^+	0.29(6)	-0.24(9)	0.00(8)	-0.01(7)
2866.6	543.7	10^+-9^+	0.26(6)	-0.02(8)	0.35(7)	0.34(6)

E_{level} KeV	E_γ KeV	$J_i^\pi - J_f^\pi$	a_2[1]	a_4[1]	δ	
					CST	LSF
					$4.0^{+1.2}_{-0.7}$	$4.2^{+1.0}_{-0.8}$
2901.2	1076.0	$11^- - 9^-$	0.49(4)	0.20(7)	0.17(11)	0.16(4)
2915.7	959.0	$10^- - 8^-$	0.35(7)	-0.01(8)	0.06(7)	$0.05^{+0.07}_{-0.0}$
3298.3	1030.6	$12^+ - 10^+$	0.39(7)	0.04(9)	0.02(8)	0.03(6)
3327.7	1005.0	$11^+ - 9^+$	0.27(10)	0.01(15)	-0.05(11)	-0.06(10)
3531.1	1180.1*	$12^- - 10^-$	0.35(8)	-0.14(12)	-0.01(8)	-0.01(7)
3628.0	1012.8	$13^+ - 11^+$	0.46(6)	0.11(8)	0.07(7)	0.06(5)
3689.2	1071.2	$12^+ - 10^+$	0.28(17)	-0.26(26)	-0.07(15)	-0.07(15)
4146.1	1244.9	$13^- - 11^-$	0.31(5)	0.08(8)	-0.05(5)	-0.06(4)
4442.1	1109.7	$13^- - 11^-$	0.24(6)	-0.30(9)	-0.11(6)	-0.11(5)
4558.0	1260.3	$14^+ - 12^+$	0.36(13)	-0.06(19)	-0.04(12)	-0.04(10)
4842.6	1311.5	$14^- - 12^-$	0.20(8)	0.06(13)	-0.15(8)	-0.15(6)
4848.3	1220.3	$15^+ - 13^+$	0.30(6)	0.10(9)	-0.04(6)	-0.09(4)
4973.2	1284.0	$14^+ - 12^+$	0.57(22)	0.09(34)	$0.12^{+0.2}_{-0.20}$	$0.12^{+0.21}_{-0.1}$
5193.4	1180.1*	$13^+ - 11^+$	0.35(8)	-0.14(12)	-0.02(7)	-0.03(7)
5446.7	1300.6	$15^- - 13^-$	0.45(9)	0.09(12)	0.09(10)	0.01(7)
5998.7	1440.1	$16^+ - 14^+$	0.45(8)	0.18(11)	0.00(7)	0.00(4)
6097.6	1255.0	$16^- - 14^-$	0.48(29)	-0.17(36)	0.02(21)	$0.02^{+0.21}_{-0.1}$
7450.2	1352.6	$18^- - 16^-$	0.27(14)	-0.20(22)	$0.00^{+0.2}_{-0.21}$	-0.13(8)
?	1457.5	15-13	0.37(15)	0.19(23)	0.02(4)	-0.04(11)

* This γ-transition has two placements.

3-C- Results calculated by the LSF-method:

The weighted averages of $\rho_2(J_i)$ presented in Table (2) were computer fitted as mentioned in section(c) . In this fitting, $\rho_2(5)$ and $\rho_2(18)$ were excluded since only one pure transition from levels with J_i=5 and J_i=18 were reported in ref. [1] and the error associated with the a_2-coefficients are relatively large. The fitting equation was as follows:

$$\rho_2(J_i) = -0.35512 + 0.01651 J_i - 0.00904 J_i^2 + 0.00027 J_i^3 \quad \ldots\ldots\ldots\ldots(8)$$

at $\chi^2_{min} = 1.5$.

The $\rho_2(J_i)$ values calculated for each J_i were then as follows:

$\rho_2(2)$ = -0.35610	$\rho_2(10)$ = -0.82402
$\rho_2(3)$ = -0.37966	$\rho_2(11)$ = -0.90799
$\rho_2(4)$ = -0.41644	$\rho_2(12)$ = -0.99220
$\rho_2(5)$ = -0.46482	$\rho_2(13)$ = -1.07506
$\rho_2(6)$ = -0.52378	$\rho_2(14)$ = -1.15494
$\rho_2(7)$ = -0.58990	$\rho_2(15)$ = -1.23022
$\rho_2(8)$ = -0.66338	$\rho_2(16)$ = -1.29928
$\rho_2(9)$ = -0.74194	$\rho_2(18)$ = -1.41226

The δ-values calculated using these $\rho_2(J_i)$ values are also presented in Table (3). The comparison of δ-values calculated by this method with those calculated by the CST-method shows that the agreement is excellent for γ-transitions from levels with

J_i=4, 6, 8, 10, 11, 12, 13, 14 and 16. The results can, therefore, be discussed as mentioned in subsection 3-B for γ-transitions from such levels.

For γ-transitions from levels with J_i=5, the δ-values calculated by the LSF-method are, within associated errors, consistent with those calculated by the CST-method. The discrepancy occurs in the case of the 418.0 KeV(5^{-}-3^{-}) transition from the 878.2 KeV level. This indicates that the a_2-coefficient reported in ref. [1] for this transition has been over-estimated.

The imaginary roots obtained in the calculation of δ-values of the 474.2 KeV(8^{+}-7^{+}) transition from the 1764.1 KeV level confirm the inaccuracy of the a_2-coefficient reported in ref. [1] for this transition.

In the case of levels with J_i=7 and 9, the δ-values calculated for γ-transitions from such levels by the LSF-method are also consistent within the associated errors with those calculated by the CST-method. According to the LSF-method the a_2-coefficient of the 628.0 KeV(7^{+}-5^{+})transition from the 1509.0 KeV level is better than that of the 680.6 KeV(7^{-}-5^{-}) transition from the 937.6 KeV level whereas the a_2-coefficient of the 680.6 KeV transition was considered to be better than that of the 623.0 KeV transition in the CST-method because the relatively large error associated with the a_2-coefficient of the 623.0 KeV transition with respect to that associated with the a_2-coefficient of the 680.6 KeV transition made the weighted average of $\rho_2(7)$ to be closer to that calculated using the a_2-coefficient of the 680.6 KeV transition.

In the case of the γ-transition from levels with J_i=18, only one transition, 1352.6 KeV(18^{-}-16^{-}), from the 7450.2 KeV level has been reported in ref. [1]. The δ-value calculated by the CST-method should, therefore, be zero as shown in Table (3). In the LSF-method, the δ-value calculated for this transition indicates that the a_2-coefficient is under-estimated, and for a pure E2 transition it should have been a_2=0.489.

D- <u>Adopted δ-values for mixed transitions</u>:

The weighted average of δ-values calculated for mixed transitions from levels of ^{80}Y are presented in Table (4) as adopted δ-values. The large errors associated with δ-values are taken into consideration.

Table (4)

Adopted δ-values for mixed transitions from levels of ^{80}Y

E_{level} KeV	E_γ KeV	$J_i^\pi - J_f^\pi$	δ			
			a_2-Ratio	CST	LSF	Adopted
257.0	257.0	5^{-}-4^{-}	-	0.06(5) $-(10^{+13}_{-4})$	0.02(4) $-(7.2^{+2.5}_{-1.4})$	0.04(3) -7.3(25)
323.6	95.5	2^{-}-1^{-}	-	-	0.04(5) -3.0(5)	-0.04(5) -3.0(5)
455.2	143.0	3^{+}-2^{+}	-	-	-0.01(3) -3.8(5)	-0.01(3) -3.8(5)
459.8	132.6	3^{-}-2^{-}	-	-	-0.04(6) $-(3.4^{+0.9}_{-0.6})$	-0.04(6) -3.4(9)
570.0	312.9	6^{-}-5^{-}	$-(0.16^{+0.22}_{-0.16})$ $-(3.2^{+\ .2}_{-1.})$	-0.02(6) $-(6.5^{+3.}_{-1.8})$	-0.02(6) $-(6.2^{+3.0}_{-1.6})$	-0.02(4) -6.3(23)

E_{level} KeV	E_γ KeV	J − J	δ			
			a_2-Ratio	CST	LSF	Adopted
648.1	192.9	4^+-3^+	0.05(9) $-(7^{+12}_{-3})$	0.05(6) $-(7.3^{+\ .3}_{-2.2})$	0.05(4) $-(7.3^{+2.7}_{-1.})$	0.05(4) -7.3(24)
663.2	203.0	4^--3^-	-0.05(12) $-(4.2^{+11.3}_{-1.6})$	-0.05(11) $-(4.1^{+3.3}_{-1.4})$	-0.05(7) $-(4.1^{+1.}_{-\ .})$	-0.05(5) -4.1(14)
878.1	215.1	5^--4^-	0.01(9) $-(6.9^{+\ .3}_{-2.})$	0.01(8) $-(6.9^{+\ .3}_{-2.})$	-0.05(6) $-(4.8^{+1.}_{-1.1})$	-0.02(4) -5.0(18)
886.0	237.9	5^+-4^-	-	0.04(7) $-(8.9^{+10.6}_{-3.2})$	0.00(4) $-(6.2^{+1.7}_{-1.2})$	0.01(3) -6.3(17)
1175.2	289.0	6^+-5^+	0.12(4)	0.11(4)	0.11(4)	0.11(2)
1185.2	299.0	6^+-5^+	0.10(5)	0.10(4)	0.10(4)	0.10(2)
1206.5	328.2	6^--5^-	-	-0.07(4) $-(4.8^{+1.3}_{-0.8})$	-0.08(3) $-(4.6^{+0.}_{-0.6})$	-0.08(2) -0.47(7)
1290.0	231.0	7^+-6^+	-	$-(0.28^{+0.13}_{-0.10})-(2.4^{+0.9}_{-0.6})$	-0.20(6) -3.1(6)	-0.21(5) -2.9(5)
1488.1	281.4	7^--6^-	-	-0.11(5) $-(4.3^{+1.3}_{-0.})$	-0.06(3) $-(5.3^{+1.0}_{-0.7})$	-0.07(3) -4.9(8)
1490.2	200.0	8^+-7^+	$-(0.61^{+?}_{-0.42})$ $-(1.3^{+1.}_{-?})$	$-(0.44^{+0.17}_{-0.10})$-1.7(4)	-0.42(7) -1.7(2)	-0.42(6) -1.7(2)
1509.0	333.7	7^+-6^+	$-(0.22^{+0.37}_{-0.22})$ $-(2.9^{+5.2}_{-\ .})$	$-(0.31^{+0.13}_{-0.10})-(2.2^{+0.8}_{-0.5})$	-0.22(5) -2.9(4)	-0.23(5) -2.8(4)
	323.9	7^+-6^+	$-(0.09^{+0.21}_{-0.17})$ $-(4.6^{+18.1}_{-2.3})$	-0.14(16) $-(3.7^{+3.9}_{-1.5})$	-0.09(12) $-(4.6^{+4.3}_{-1.6})$	-0.10(9) -4.1(28)
1823.7	335.5	9^+-8^+	$-(0.17^{+0.13}_{-0.11})$ $-(3.6^{+2.5}_{-1.2})$	-0.32(10) -2.3(6)	-0.26(5) -2.7(4)	-0.26(4) -2.6(3)
1956.7	468.6	8^--7^-	0.06(5) $-(21^{+\infty}_{-10})$	0.05(6) $-(17^{+\infty}_{-12})$	0.06(5) $-(17.9^{+\infty}_{-8.4})$	0.06(3) $19^{+\infty}_{-6}$
2305.0	348.4	9^--8^-	-	-0.06(6) $-(6.2^{+}_{-\ 7})$	-0.03(5) $-(7.3^{+}_{-})$	-0.04(4) -6.7(24)
2615.2	347.0	11^+-10^+	$-(0.25^{+0.15}_{-0.12})$ $-(2.9^{+1.5}_{-0.9})$	-0.15(7) $-(4.1^{+1.}_{-1.0})$	-0.15(4) $-(4.2^{+0.9}_{-0.})$	-0.16(3) -3.9(7)
2866.6	543.7	10^+-9^+	-	0.35(7) $4.0^{+1.2}_{-0.7}$	0.34(6) $4.2^{+1.0}_{-0.8}$	0.34(5) 4.1(8)

4- CONCLUSION

The δ-values of γ-transitions from low and high-spin states populated in the ^{58}Ni(^{24}Mg,pnγ)^{80}Y reaction have been calculated in the present work using the a_2-ratio, CST and LSF-methods and the experimental a_2-coefficients reported in ref. [1]. Although no δ-value was reported for any γ-transition in ref. [1], the good agreement between the δ-values calculated by the three method for most of the γ-transitions confirm the validity of these methods for calculating the δ-values of γ-transitions from high-spin states. The weighted averages of the δ-values calculated for mixed γ-transitions may, therefore, considered as adopted δ-values for such transitions.

Furthermore, all the three methods depend upon the experimental data only and are rather simple. An ordinary modern calculator is quite enough to perform all the necessary calculations by the a_2-ratio and CST methods and a personal computer for the LSF-methods.

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