Mathematical model of the dynamics (changes) of the temporal structures using Fokker- Plank Equation

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ملخص

من المعروف، أن معادلة فوكر- بلانك تصف ديناميكا (تغيرات) تعديلات كثافة الاحتمالية المنسابة، وهذا مبين في خلاصة فضاء الطور متعدد الأبعاد . والموديلات الرياضية الممثلة للهياكل الوقتية،والتي يتناولها هذا البحث ، تسمح لنا، تحليليا، بتحديد تعديلات فضاء الطور لهذه الهياكل (أو البنى) والتي يمكن توظيفها لتقليص الأضطرابات و الضوضاء في الإشارات .

SUMMARY

It is known, that the Fokker-plank Equation (FPE) describes dynamics (changes) of modifications of probabilities density in many-dimensional phase space. The represented mathematical models of dynamics of temporal structures, allow to determine analytical modifications phase coordinates of these structures of perturbations and noise operations.

Introduction:

Synchronization phenomena been a topic of scientific research for many years[1]and in many systems, ranging from physics to biology[2] . Many different situation have been considered, including synchronization of limit cycle oscillators[3,4], synchronization of chaotic systems[5] , partial(i.e., phase) synchronization[6],generalized synchronization[7,8], synchronization

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of stochastic systems[9], systems characterized by a purely temporal dynamics. synchronization phenomena have also been studied in systems with spatial degrees of freedom in par synchronization particular, synchronization of two spatiotemporally chaotic fields[11,12].In Refs.(11,12) the two chaotic fields are taken as the two independent components of a vector field, as for example, the two polarization components of an electric field vector. Synchronization is mediated by the dynamics of spatially localized vectorial structures. In this paper we address the question of the dynamical of temporal structures of perturbation and noise operations. This regime is characterized by the fact that local perturbations are advocated more rapidly than their rate of spreading. Therefore, macroscopic patterns named noise-sustained structures emerge in this regime if noise is present at all time. Optical noise-sustained structures [13] have been predicted in optical parametric oscillators (OPO`s) in which the nonlinear crystal is pumped by a field linearly polarized with frequency f_o (signal and idler) are orthogonally polarized. Our mathematical models of dynamics of temporal structures, allow determining analytical modifications phase coordinates of these structures of perturbations and noise operations.

Theory:

Let consider dynamics (changes) of a system; its behavior is described by the following differential equation:

$$\frac{dT_i}{dt} + f(T_1, T_2, ..., T_n) = \xi_i \qquad (i = 1,2......n) \qquad (1)$$

Ti –is the phase coordinates, for example, the realization of research durations temporal intervals; f_i (…) describes power performances of a phase space that influencing the modification of this time intervals, and ζ_i is a random disturbance, operate on the given temporal structure and changes of phase coordinates of the given temporal structure, or it may be the exterior noise with a known matrix of spectral densities S= $\| S_{ik} \|$, where (k=1,2,…,n).Further we consider, that initial values phase coordinates T_1 (0),T_2 (0),…, T_n (0) are considered casual and their distribution characterizes a probability density[14].

$$P(T_1,T_2,......,T_n,0)=P_o(T_1,T_2,......) \qquad (2$$

Which should satisfy the condition of normalization?

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p_o dT_1 dT_2....dT_n = 1 \qquad (3)$$

The equation FPE allows basically to place (install) dynamic linking of the following probability density P ($T_1,T_2,\ldots,T_n$) with a performance of a temporal structure Fi ($T_1,T_2,\ldots,T_n$) as follows[15].

$$\frac{\partial In(p)}{\partial t}-\sum_{i=1}^{n} f_i \frac{\partial In(p)}{\partial T_i}-\frac{1}{2}\sum_{i,j=1}^{n} S_{ik}\frac{\partial^2 In(p)}{\partial T_i \partial T_j}+\frac{\partial In(p)\partial In(p)}{\partial T_i \partial T_j}=\sum_{i=1}^{n}\frac{\partial f(T,t)}{\partial T_i} \qquad (4)$$

Under certain condition the durations of time intervals T_i can be proportional to performances of a phase space power, expressed through F_i . if it is limited only to these superposition's and to neglect exterior perturbations and noise, the equation of dynamics (changes) can be noted as a system of two equations:

$$\frac{\partial In(p(T,t))}{\partial t}-f(T,t)\frac{\partial In(p(T,t))}{\partial T}=\frac{\partial f(T,t)}{\partial T} \qquad (5)$$

$$\frac{dT}{dt}+f\ (T,\ t)\ =\ 0$$

If the distribution density of the durations of time intervals dose not vary in time, the first term in the equation (4) is equal to 0, and then we get:

$$\frac{\partial}{\partial T}[In\, f\,(T,t)+In\, p\,(T)]=\frac{\partial}{\partial T}In[f\,(T,t)\,p\,(T)]=0\ , \qquad (6)$$

where, $In\, f\,(T,t) + In\, p\,(T) = \varphi(T)$

The function $\varphi(T)$ is determined from the entry conditions . For example, when, $T=T_o$

$f\,(T,\,t) = f_o\,(T_o,t)$,in particular, if

$$\sum_{i=1}^{n} k_i(t) \quad \text{- degree function}, f_o(T_o, t_o) =$$

$$\varphi(t) = \text{In}\,[f_0(T_0, t)\, p\,(T_0)],$$

When performance of a temporal structure required, the time intervals will look like the following:

$$f(T,t) = \left|f_0(T_0, t)\;\; P(T_0)/\,P(T)\right| \tag{7}$$

Substituting expression (7) in (4) and taking this system rather T(t),we shall have the following equation for the temporal dynamics(changes) of intervals;

$$\int_0^{\infty} p(T)\,dT = -\,P(T_0)\int_0^{\infty} f_0(T_0, t)\,dt + C_1 \tag{8}$$

Where C_1, is a constant of integration, which is defined from the entry conditions. For example, when $t{=}t_o$ and $T{=}T_o$, we obtain, that

$$C_1 = \int_0^{\infty} P\,(T)\,dT\Big|_{T=T_0} + P\,(T_0)\int_0^{\infty} f\,(T_0, t)\,dt\Big|_{t=t_0} \tag{9}$$

In this case the probability distribution p (T,t) varies in time, and the common solution is possible to be as

$$f(T,t) =$$

$$[1/p(T,t)]\left[f(T_0,t)\,p(T_0,t) - \int_0^{\infty}\frac{\partial p(T,t)}{\partial t}\,dT\Big|_{T=T_0} + \int_0^{\infty}\frac{\partial p(T,t)}{\partial t}\,dT\right] \tag{10}$$

For the case of the normal law of distribution we have

$$p(T)=\left[1/(\sqrt{2\pi\sigma})\,s\right]\exp\left[-1/2(T-m/\sigma)\,I\right],$$

s and I are constants factors.
When p (T,t)= p (T) and f (T,t) = K_o - do not vary in time, the performance of a temporal structure will look like the following:

$$f(T)=K_n\exp\left[1/2\,(T-m/\sigma)^2\right],\ \text{where} \qquad (11)$$
$$K_n=K_0\exp\left[1/2\,(T_0-m/\sigma)^2\right].$$

For more spreaded case, when the distribution p(T) has asymmetry and kurtosis E_x (that is two model) we shall have, that p (T) is possible to present as a sum of two normal distributions

$$p(T)=g_1(1/\sqrt{2\pi\sigma_1{}^2})\exp\left[-1/2(T-m_1/\sigma_1)^2\right]+$$

$$g_2(1/\sqrt{2\pi\sigma_2{}^2}\exp\left[-1/2(T-m_2/\sigma_2)^2\right] \qquad (12) \quad +$$

In this equation the factors g_1, g_2, σ_1 and σ_2 are determined by the numerical methods of smoothing of stochastic serieses for average values (m_1,m_2),and also for known values of first four moment of casual distribution. Then, with allowance for equation (12) of dynamics (changes) of a modification of temporary duration intervals T(t) is possible to be as

$$\frac{dT}{dt}+\frac{K_0P(T_0)}{\left[g_1\left(1/\sqrt{2\pi\sigma_1^2}\right)e^{-1/2(T-m_1/\sigma_1)^2}\right]+g_2\left(1/\sqrt{2\pi\sigma_2^2}e^{-1/2(T-m_2/\sigma_2)^2}\right)} \qquad (13$$

In specific case, when p(T) varies linearly with a modification of duration T (t),we shall receive the following expression for p (T):

$$P\ (T)= IT+ P \qquad (14)$$

Where I and P are constants factors.

Then we shall have, that

$$\frac{dT}{dt} + k_0(IT_0 + P)/(IT + P) = 0 \qquad (15)$$

Integrating this equation we get

$$\frac{IT^2}{2} + PT = -K_0(IT_0 + P)\,t + C_2 \qquad (16)$$

C_2, is a new constant of integration .Equation (16) can be used for small time interval of linear approximation p(T) of an arbitrary type . Thus we can have analytical expression for an association T(t) as nonlinear function . It is necessary to mark what dynamic is (change) to investigate by a very large amount of time intervals on the basic solution of the equations FPE. the correlation between entering and exit pupils of a research system is not known[16].

Allowance model for perturbation

Let's consider now dynamics of a temporal structure with allowance for perturbation and noise operations. Accordingly let

$$\frac{z}{2}\left[\frac{\partial^2 \mathrm{Inp}}{\partial A^2} + \left(\frac{\partial \mathrm{Inp}}{\partial A}\right)^2\right] = Q(A) \qquad ; \qquad \frac{\partial \mathrm{Inp}}{\partial A} = R(A)$$

Then for p(A,t) = p(A) we have

$$\frac{\partial f}{\partial A} + f\,R(A) = -Q(A)$$

The solution form of such equation will be

$$f(A,t) = e^{-\int R(A)dA}\,(C_3 - \int Q(A)\,e^{\int R(A)dA}\,dA) \qquad (17)$$

For Q = 0 , we will have an obvious relation as in case of a lack of perturbation and noise

$$f(A,t) = e^{-\int \frac{\partial \ln p}{\partial A} dA}\, C_3 = C_3/p, \quad \text{where } f(A,t)p(A_0) = C_3 \qquad (18$$

$$\text{if} \qquad A = A_0 \quad , \quad f\,(A,t) = f\,(A_0,t)$$

$$f\,(A\,,t\,) = f\,(A_0,t)\;p(A_0)/\,p(A)$$

Substituting a value of Q(A) and R(A), in (17). We will have the following equation of dynamics (changes) of a temporal structure:

$$\frac{dA}{dt} + \frac{1}{p}\left\{C_3 - \frac{1}{2}\int Z\left[\frac{\partial^2 \ln p}{\partial A^2} + \left(\frac{\partial \ln p}{\partial A}\right)^2\right] P\,dA\right\} = \xi$$

For a common case piecewise linear approximation p (A) we have for p(A)=mA+ n

$$\frac{dA}{dt}+\frac{C_3}{mA+n}=k \qquad (19)$$

Integrating this equation, we will get

$$\frac{A}{k}+\frac{C_4}{k^2m}\ln(kmA+kn-C_4)=t+C_3 \qquad (20)$$

When k- value of perturbations is 0 at $t<0$ we get

$$C_4=k_0\,(MA_0+n);$$

$$C_3=\frac{A_0}{k}+\frac{C_4}{k^2m}\ln(kmA_0+kn-C_4)$$

For a numerical solution of this equation of dynamics it's necessary to know a value of scale factor of perturbation T_0. Scale factor (correspondence between a unit of measurements of revolting action and unit of measurements A) is defined on static performance , which divide on some linear plots (sites) .If the distribution p(A) can be described by expression of an aspect

$$P(A)=k_1\,e^{m(x-a)}$$

That

$$f(A,t)=(C_5/K_1e^{m(A-a)})-Sm/2$$

The equation of dynamics will look like the following

$$\frac{dA}{dt}+B\,e^{-m(A-a)}-\frac{Sm}{2}=d$$

Solution of this last equation is possible to be

$$A(t)=\frac{1}{m}\ln\frac{1}{C_5}\left[e^{C_6m(t+C_6)}+B\right]+a$$

integration. Thus, the represented above mathematical Where C_5 and C_6 are constants of
models of dynamics of temporal structures, allow to determine analytically modifications phase coordinates of these structures for perturbations and noise operations.

Conclusion:

It's found that the mathematical model of dynamic of temporal structures which
by the relation

$$A(t) = \frac{1}{m} \ln \frac{1}{C_5} \left[e^{C_6 m(t+C_6)} + B \right] + a$$

allow to determine analytically modifications phase coordinates of those structures for perturbation and noise operations.

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